# Singular perturbations in a reaction-diffusion model for yeast biofilm formation

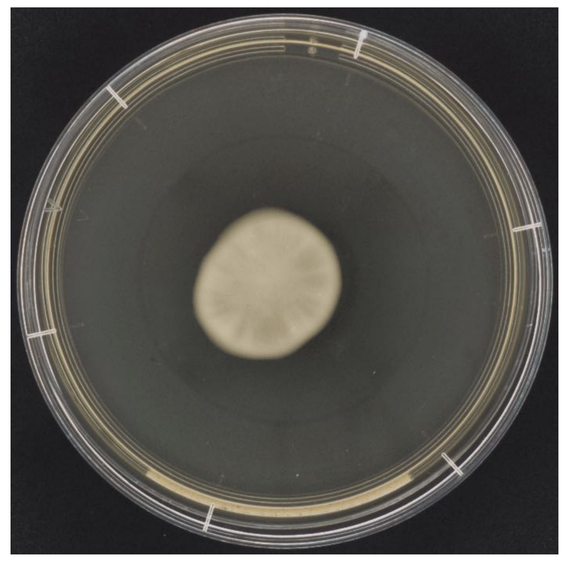
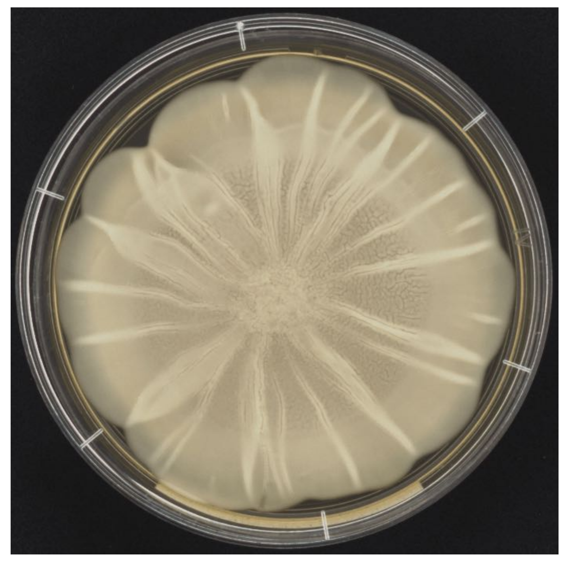

Alex Tam, Ben Binder, Ed Green, Sanjeeva Balasuriya

University of Adelaide, School of Mathematical Sciences

Email: alexander.tam@adelaide.edu.au


## Yeast biofilms

- Complex communities of yeast cells and viscous fluid.
- Mechanisms of pattern formation not fully understood.

## Mathematical model

- We investigate whether nutrient-limited growth alone can explain the floral pattern.

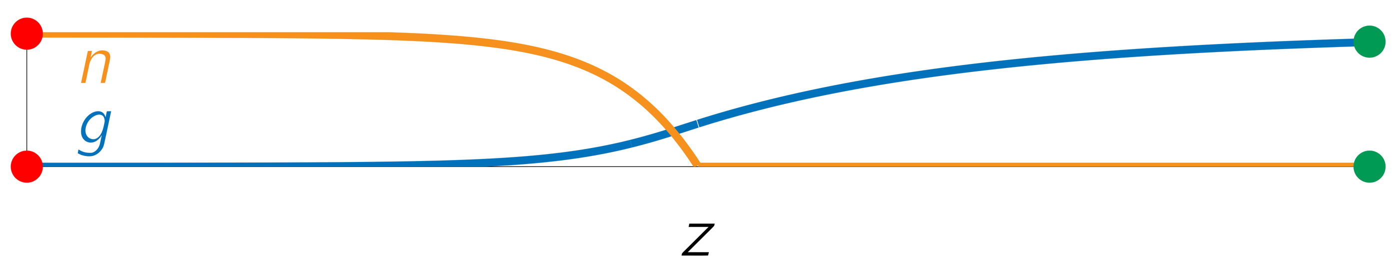
$$\frac{\partial n}{\partial t} = D\frac{\partial}{\partial r}\left(n^k \frac{\partial n}{\partial r}\right) + ng, \qquad \frac{\partial g}{\partial t} = \frac{\partial^2 g}{\partial r^2} - ng.$$

- Problem: Diffusion coefficient $D$ cannot be measured.

## Travelling wave analysis

- Let $D = \epsilon \ll 1$, $z = r - vt$, and $\zeta = \int_0^z n^{-k}\, ds$.

$$\frac{dN}{d\zeta} = \frac{1}{\epsilon}\left(v - vN - W - vG\right),$$
$$\frac{dG}{d\zeta} = WN^k,$$
$$\frac{dW}{d\zeta} = N^{k+1}G - vWN^k.$$



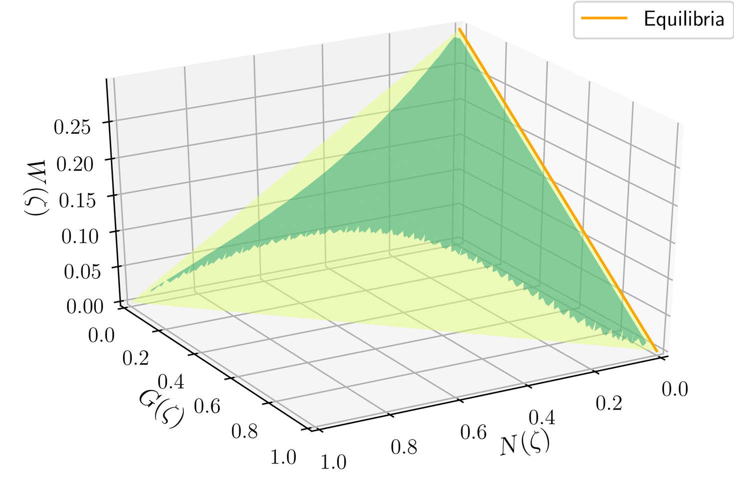

## Geometric singular perturbation theory

- Introduce re-scaled variable $\gamma = \zeta/\epsilon$ and take $\epsilon \to 0$.

Layer problem ◄— GSPT —► Reduced problem

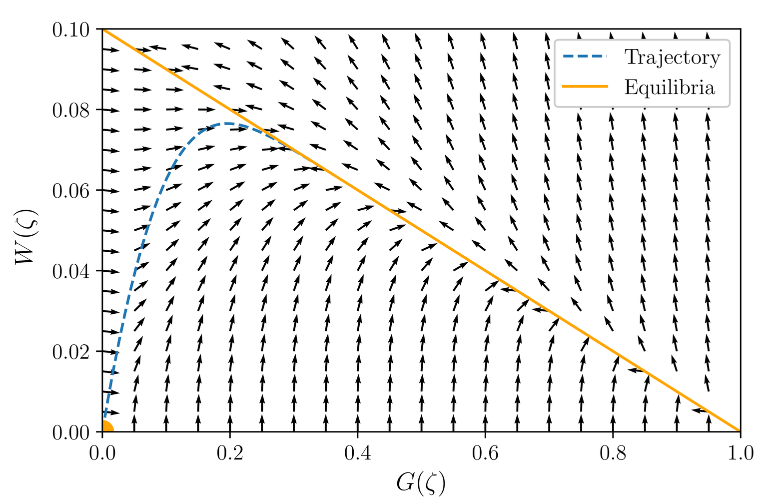
$$\frac{dN}{d\gamma} = v - vN - W - vG, \qquad 0 = v - vN - W - vG,$$
$$\frac{dG}{d\gamma} = 0, \qquad \frac{dG}{d\zeta} = WN^k,$$
$$\frac{dW}{d\gamma} = 0. \qquad \frac{dW}{d\zeta} = N^{k+1}G - vWN^k.$$

## Layer problem




$$\frac{dN}{d\gamma} = v - vN - W - vG,$$
$$\frac{dG}{d\gamma} = \epsilon WN^k,$$
$$\frac{dW}{d\gamma} = \epsilon\left(N^{k+1}G - vWN^k\right).$$

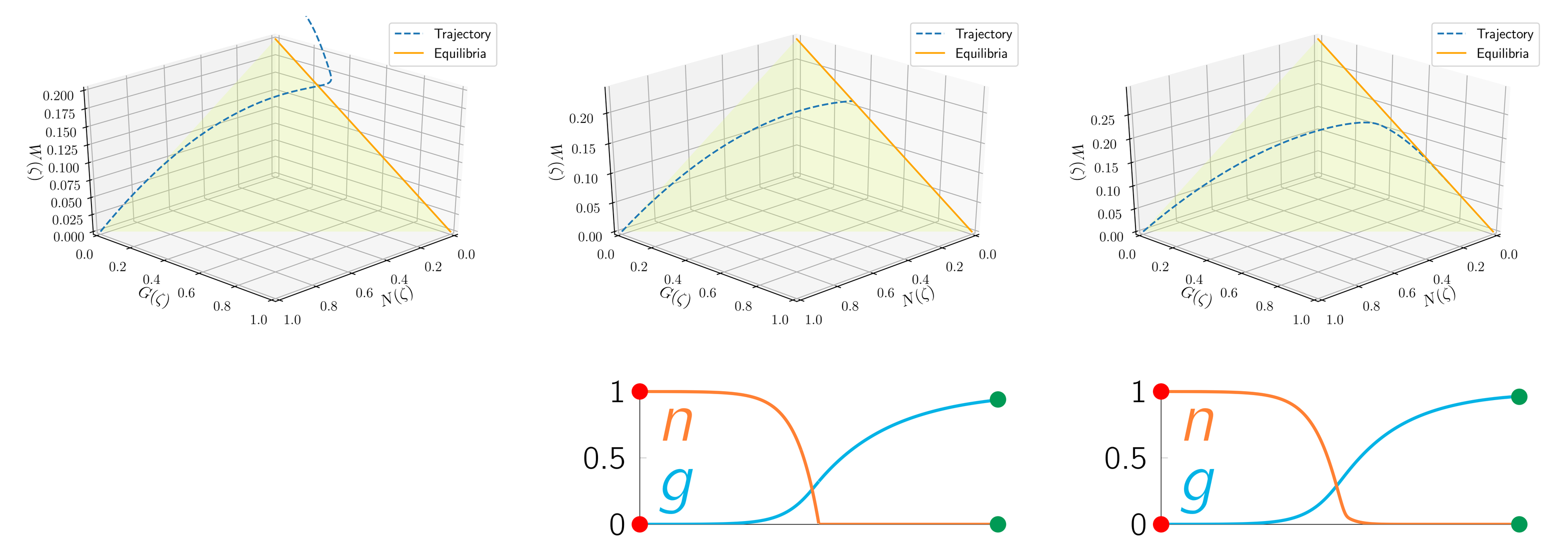
## Reduced problem



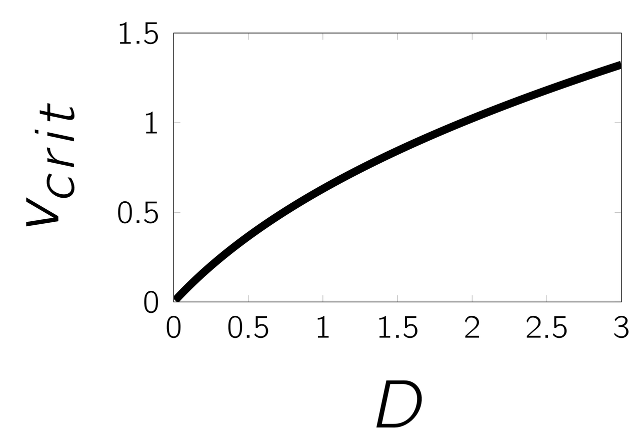

$$\frac{dG}{d\zeta} = W\left(1 - \frac{W}{v} - G\right)^k,$$
$$\frac{dW}{d\zeta} = \left(1 - \frac{W}{v} - G\right)^{k+1} G - vW\left(1 - \frac{W}{v} - G\right)^k.$$

## Numerical solutions (D = 0.3, k = 1)

| $v = 0.2$ | $v = 0.2367$ | $v = 0.3$ |
|---|---|---|

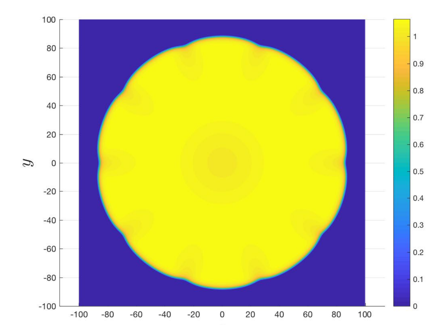
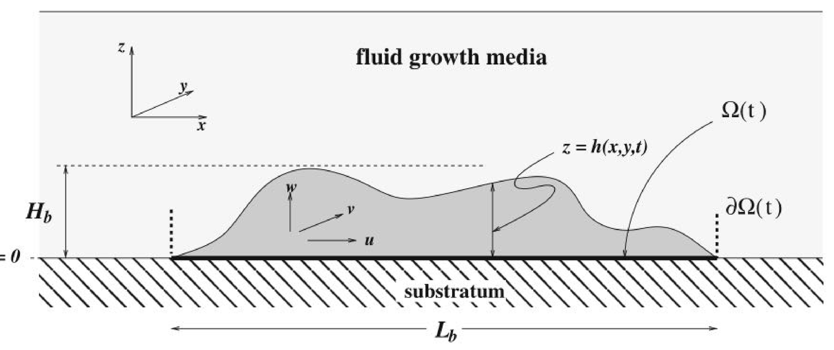



- There is a unique critical wave speed for each $D$.
- We can measure this experimentally to find $D$.




## Further work

- 2D model: stability analysis, numerical solutions; experiments; image processing; thin-film fluid modelling [1].



### References

[1] A. Tam et al. "Nutrient-limited growth with non-linear cell diffusion as a mechanism for pattern formation in floral yeast biofilms". *Journal of Theoretical Biology* (2017). Submitted.