Predicting channel bed topography in hydraulic fall experiments

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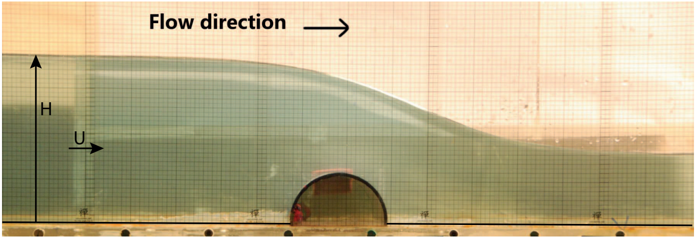
Hydraulic fall

- Waveless flow upstream and downstream
- Characterised by upstream Froude number

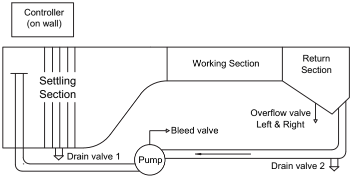
$$F = \frac{U}{\sqrt{gH}} = \frac{Q}{WH\sqrt{gH}} \quad \text{(flow rate } Q = WHU\text{)}$$

- Weakly nonlinear and fully nonlinear models
- Forward and inverse methods



Experimental method

- Closed-loop water channel
- Pump frequency controls flow rate, Froude number



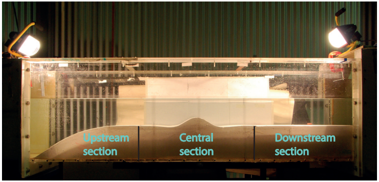
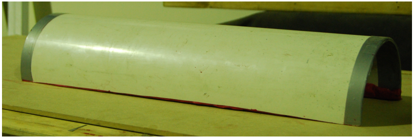
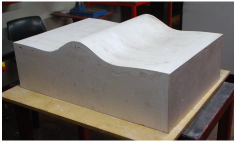


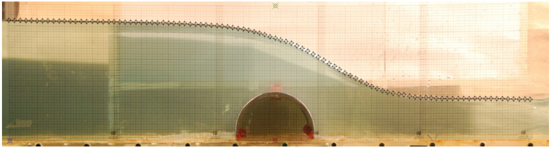
Figure : Water channel

Experimental method

- Semicircular and Gaussian topography
- Nikon D40X used to photograph steady hydraulic falls
- Spotlights, grids, dye enhance visualisation
- Free surface profiles $(\hat{x}_j, \hat{y}_j)$ extracted from images







Potential flows

- Steady flow
- Irrotational flow: $\mathbf{v} = (u, v) = \nabla\phi$
- Incompressible fluid: $\nabla.\mathbf{v} = 0$

Dimensionless model equations:

$$\phi_{xx} + \phi_{yy} = 0 \quad \text{for} \quad \sigma < y < 1 + \eta$$

$$\phi_y = \phi_x \eta_x \quad \text{on} \quad y = 1 + \eta$$

$$\frac{1}{2}(\phi_x^2 + \phi_y^2) + \frac{1}{F^2} y = \frac{1}{2} + \frac{1}{F^2} \quad \text{on} \quad y = 1 + \eta$$

$$\phi_y = \phi_x \sigma_x \quad \text{on} \quad y = \sigma$$

$$\phi \to x \quad \text{and} \quad y \to 1 \quad \text{as} \quad x \to -\infty$$

Weakly nonlinear model

- Forced Korteweg-de Vries (KdV) equation

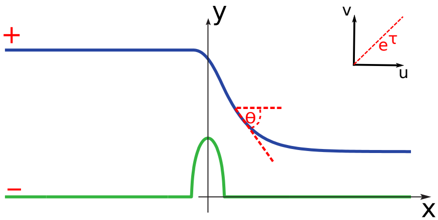
$$\eta_{xx} + \frac{9}{2}\eta^2 - 6(F-1)\eta = -3\sigma$$

Forward problem – unknown F and η

$$\frac{\mathrm{d}^2\eta}{\mathrm{d}x^2} + \frac{9}{2}\eta^2 - 6(F-1)\eta = -3\sigma_E$$

Inverse Problem – unknown σ

$$\sigma = 2(F_E - 1)\eta_E - \frac{1}{3}\frac{\mathrm{d}^2\eta_E}{\mathrm{d}x^2} - \frac{3}{2}\eta_E^2$$



- Model based on Binder, Blyth, and McCue (2013)

$$e^{2\tau^{+}} + \frac{2}{F^2} y^{+} = 1 + \frac{2}{F^2}, \qquad \tau = \log |\mathbf{v}|$$

$$\tau^{\pm}(s) = \int_{-\infty}^{\infty} \frac{\theta^{-}(t)}{1 \pm e^{\pi(s-t)}} - \frac{\theta^{+}(t)}{1 \mp e^{\pi(s-t)}} \, dt$$

$$x^{\pm}(s) = x^{\pm}(-\infty) + \int_{s}^{\infty} e^{-\tau^{\pm}(t)} \cos \theta^{\pm}(t) \, dt$$

$$y^{\pm}(s) = y^{\pm}(-\infty) + \int_{s}^{\infty} e^{-\tau^{\pm}(t)} \sin \theta^{\pm}(t) \, dt$$

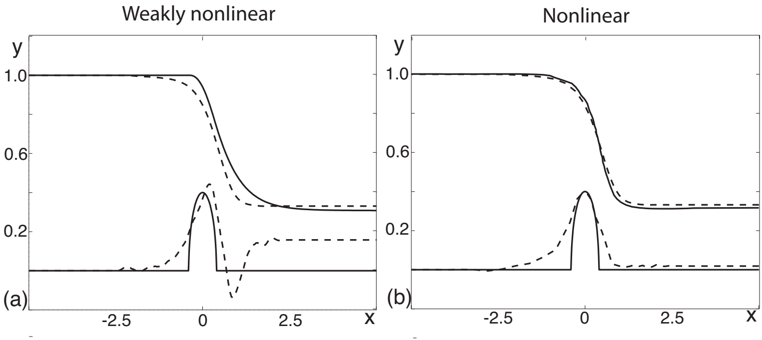


Figure : Solid curves: forward problem. Broken curves: inverse problem. $F_E = 0.38$. (Tam et al. 2015)

- Forward nonlinear solution from Forbes (1988)

Semicircle results

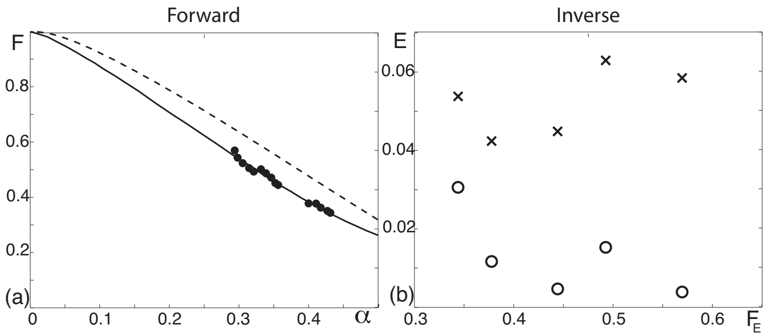


Figure : (a) Nonlinear (solid), weakly nonlinear (broken), and experiment (markers). (b) $E = |M - \alpha|$, Nonlinear (circles), weakly nonlinear (crosses)

- Weakly nonlinear model over-estimates Froude number
- Both models accurately predict maximum topography height

Gaussian results

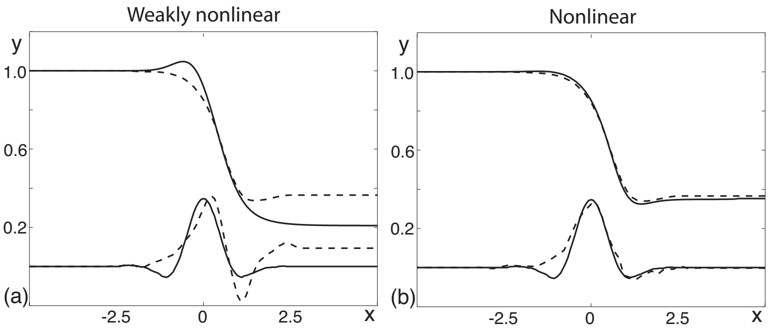


Figure : Solid curves: forward problem. Broken curves: inverse problem. $F_E = 0.45$.

- Smooth topography reduces error at bump supports
- Recirculatory regions not predicted by potential flow model

Conclusion

- Weakly nonlinear (KdV) and nonlinear models
- Inverse methods used to predict topography in experiments
- Both models accurately predict maximum topography height
- Nonlinear model accurately predicts topography shape

References

Binder, B.J., Blyth, M.G., and McCue, S.W. 2013, "Free-surface flow past arbitrary topography and an inverse approach for wave-free solutions", *IMA Journal of Applied Mathematics*, vol. 78, no. 4, pp. 685–696.

Forbes, L.K. 1988, "Critical free-surface flow over a semi-circular obstruction", *Journal of Engineering Mathematics*, vol. 22, no. 1, pp. 3–13.

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